Sum and Difference Identities

Read through the following identities and make sure you can interpret what they mean. These identities are referred to as the **sum and difference** identities:

$$sin(u+v) = sin\,u\,cos\,v + cos\,u\,sin\,v$$

$$sin(u-v) = sin\,u\,cos\,v - cos\,u\,sin\,v$$

$$cos(u+v) = cos\,u\,cos\,v \ - \ sin\,u\,sin\,v$$

$$cos(u-v) = cos\,u\,cos\,v \ + \ sin\,u\,sin\,v$$

$$tan(u+v) = \frac{tan\,u + tan\,v}{1 - tan\,u\,tan\,v}$$

$$tan(u-v) = \frac{tan\,u - tan\,v}{1 + tan\,u\,tan\,v}$$

How can you use the identities above to find the exact value of $sin\,105^o$

$105^\circ = 60^\circ + 45^\circ$

$sin(60^\circ + 45^\circ) = sin\,60 \cdot cos\,45 + cos\,60 \cdot sin\,45$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{3}}{4}$

Use an identity to find the exact value of $cos\,75^o$.

$$75^0 = 45^0 + 30^0$$

$$\cos 75 = \cos 45 \cdot \cos 30 - \sin 45 \cdot \sin 30$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

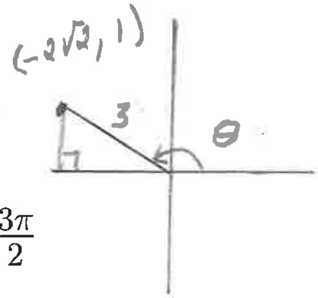
Use an identity to find the exact value of $tan\,\frac{11\pi}{12}$.

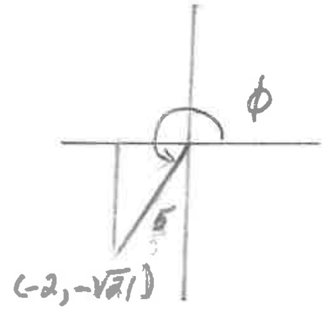
$$\frac{11\pi}{12} = \frac{9\pi}{12} + \frac{2\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{6}$$

$$\tan\frac{11\pi}{12} = \frac{\tan\frac{3\pi}{4} + \tan\frac{\pi}{6}}{1 - \tan\frac{3\pi}{4} \cdot \tan\frac{\pi}{6}}$$

$$= \frac{1 + \frac{\sqrt{3}}{3}}{1 - 1 \cdot \frac{\sqrt{3}}{3}} = \frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}} = \frac{3+\sqrt{3}}{3-\sqrt{3}}$$

If $sin\,\theta = \frac{1}{3}$ and $\frac{\pi}{2} < \theta < \pi$
and
$cos\,\phi = -\frac{2}{5}$ and $\pi < \phi < \frac{3\pi}{2}$





find $cos(\theta - \phi)$ $= \cos\theta\cos\phi + \sin\theta\sin\phi$

$\cos\theta = -\frac{2\sqrt{2}}{3}$, $\sin\phi = \frac{-\sqrt{21}}{5}$ Now substitute

$$\cos(\theta-\phi) = \left(-\frac{\sqrt{2}}{3}\right)\left(-\frac{2}{5}\right) + \left(\frac{1}{3}\right)\left(\frac{-\sqrt{21}}{5}\right)$$

$$= \frac{2\sqrt{2} - \sqrt{21}}{15}$$

find $tan(\theta + \phi)$ $= \dfrac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$

$\tan\theta = \frac{-1}{2\sqrt{2}}$,

$\tan\phi = \frac{\sqrt{21}}{2}$

$$\tan(\theta+\phi) = \frac{\frac{-1}{2\sqrt{2}} + \frac{\sqrt{21}}{2}}{1 - \left(\frac{-1}{2\sqrt{2}}\right)\left(\frac{\sqrt{21}}{2}\right)}$$

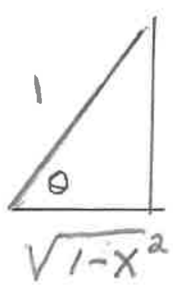
Use a sum or difference identity to verify the identity
$tan(\pi + x) = tan\, x$

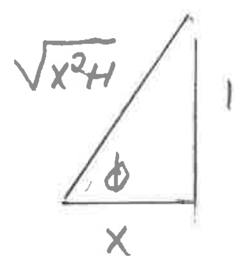
$$\tan(\pi + x) = \frac{\tan\pi + \tan x}{1 - (\tan\pi)(\tan x)}$$

$$= \frac{0 + \tan x}{1 - 0} = \tan x$$

Express $cos\,(sin^{-1}x - cot^{-1}x)$ as an algebraic function of x.

Let $\theta = \sin^{-1} x$, $\phi = \cot^{-1} x$





$$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$

$$= \frac{\sqrt{1-x^2}}{1} \cdot \frac{x}{\sqrt{x^2+1}} + \frac{x}{1} \cdot \frac{1}{\sqrt{x^2+1}}$$

$$= \frac{x\sqrt{1-x^2} + x}{\sqrt{x^2+1}}$$